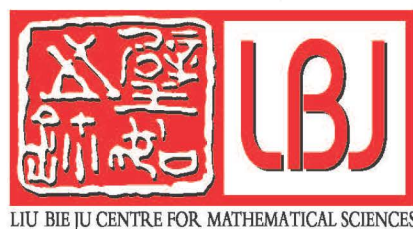




School of Data Science

香港城市大學  
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# Approximation Numbers of Embeddings of Anisotropic Sobolev Spaces of Dominating Mixed Smoothness

**DATE** 3 July 2019 (Wednesday)  
**TIME** 4:00pm to 5:00pm  
**VENUE** P7510, 7/F, Yeung Kin Man Academic Building (YEUNG),  
City University of Hong Kong

## Abstract

We investigate the approximation of  $d$ -variate periodic functions in anisotropic Sobolev spaces of dominating mixed (fractional) smoothness  $\vec{s}$  on the  $d$ -dimensional torus, where the approximation error is measured in the  $L_2$ -norm.

As it is well-known, in high dimensions functions from isotropic Sobolev spaces  $H^s(\mathbb{T}^d)$  can not be approximated sufficiently fast (in the sense of approximation numbers of corresponding embeddings). One needs to switch to smaller spaces. Since the beginning of the sixties it is known that Sobolev spaces of dominating mixed smoothness  $H_{\text{mix}}^s(\mathbb{T}^d)$  may help. However, for very large dimensions even these classes are oversized. A way out is to sort the variables in dependence of their importance (in our case in dependence of the smoothness). We associate to each variable different smoothness assumptions. As smoother the function is with respect to the variable  $x_\ell$  as weaker is the influence of this variable. This philosophy is reflected in the choice of the function space  $H_{\text{mix}}^{\vec{s}}(\mathbb{T}^d)$  characterized by the norm

$$\|f\|_{H_{\text{mix}}^{\vec{s}}(\mathbb{T}^d)} := \left[ \sum_{k \in \mathbb{Z}^d} |c_k(f)|^2 \prod_{j=1}^d (1 + |k_j|)^{2s_j} \right]^{1/2} < \infty.$$

We assume  $\vec{s} = (s_1, s_2, \dots, s_d)$  and

$$s_1 = s_2 = \dots = s_\nu < s_{\nu+1} \leq s_{\nu+2} \leq \dots \leq s_d$$

for some number  $1 \leq \nu < d$ . It will be the main aim of my talk to describe the behaviour of the approximation numbers

$$a_n(I_d : H_{\text{mix}}^{\vec{s}}(\mathbb{T}^d) \rightarrow L_2(\mathbb{T}^d))$$

in dependence of  $n, \vec{s}, \nu$  and  $d$ . Almost all of our results will be based on an elementary lemma, simplifying in this way also our earlier results with respect to this topic.

This is joint work with Thomas Kühn (Leipzig) and Tino Ullrich (Bonn).

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